

On the Movement of Briquetted Mass in Extruder. Exact Solutions. Message 1.

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Abstract— The increased interest in cold agglomeration in recent years has been largely due to the successful experience in operating briquette lines based on stiff vacuum extrusion (SVE). The high performance of SVE extruders and the satisfactory metallurgical properties of extrusion briquettes obtained in this way (brex) make it possible to consider this cold agglomeration technology as promising. SVE Extruders allow efficient briquetting of the materials with moisture contents values in the range of 12–16% and compacting pressure of 3.5–4.5 MPa, leading to the possibility of achieving high values of mechanical strength of raw briquettes and eliminates the need for drying briquetted charge and heat treatment of the green briquettes. The growing scale of practical use of extruders in the steel industry necessitated the development of simple and effective methods for determining their operating parameters. The briquetted mass is a moistened plastic mass, driven by the blades of a rotating auger and squeezed out further through the holes in the extruder die in the form of elongated briquettes, repeating in cross section the shape of the hole. In application to the optimization problems of extrusion briquette technology, the exact solution of the Navier–Stokes equations for a viscous incompressible medium shifted between coaxial cylinders along the common axis of symmetry and twisted around it by longitudinal displacement and axial rotation of the inner cylinder is given, respectively, under sticking conditions and given longitudinal pressure drop. In particular, it was found that the speed of transportation of the mixed mass cannot exceed the speed delivered by the supplied pressure, and the formula of the latter is transferred to the compressible medium as a special viscosity integral for a viscoplastic medium, where it serves as a generalization of known exact solutions. A similar solution for a compressible medium is being sought. The obtained analytical dependencies can be used to calculate the parameters of industrial briquette extruders operating in both the stiff extrusion mode and the semi-stiff and soft extrusion mode, differing in the moisture values of the briquetted mass and the values of the applied pressure.

Keywords: stiff vacuum extrusion, briquetting, brex, viscous incompressible medium, Navier–Stokes equations, coaxial cylinders, spiral Couette–Poiseuille flow

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Stiff vacuum extrusion (SVE) is widely used for the production of metallurgical briquettes—components of the burden of blast furnaces and ferroalloy furnaces [1–8]. The mechanical and hot strength of extrusion briquettes (brex) is ensured by a smaller amount of binder materials than in alternative briquette technologies (roller briquetting and vibropressing), which, together with higher extruder productivity, makes it possible to consider that SVE fully meet the criteria for the best available technologies in ferrous metallurgy (BAT, [9]).

Unlike roller briquetting and vibropressing, SVE allows to agglomerate moistened materials with a moisture content in the range of 12–16% (maximum up to 20%) at pressures from 2.5 MPa to 4.5 MPa.

Varieties of extrusion agglomeration also used in metallurgy are soft extrusion (moisture content 10–27%, pressure 0.4–1.2 MPa) and semi-stiff extrusion (moisture content 15–22%, pressure 1.5–2.2 MPa). The most important criterion for the applicability of SVE for producing briquettes is the plasticity of the moldable mixture, which makes it possible to move such a mixture with the blades of a rotating screw and compact it when it pushed through holes in the extruder dies.

Due to the rotation of the auger blades in the working chamber of the extruder, the molded mass performs translational and rotational motion, which is slowed down by the walls of the housing (Fig. 1). The extruder can be represented in the form of two coaxial

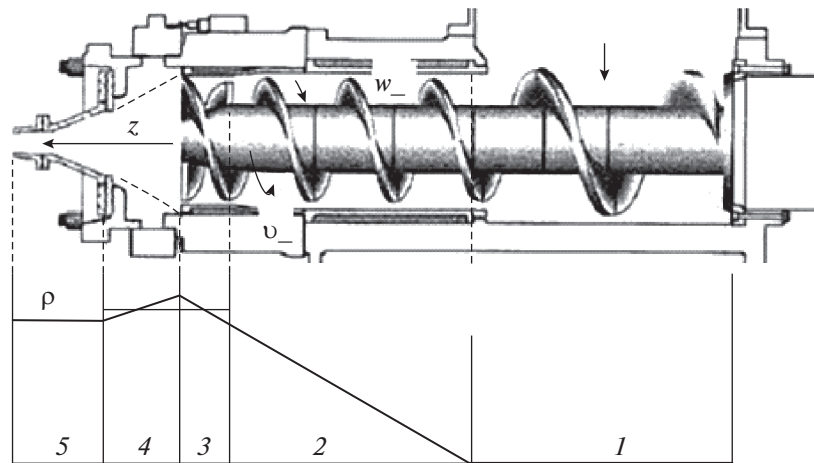


Fig. 1. Stages of compaction in the working area of the extruder. (1) feed zone without compaction, (2) compression zone, (3) homogenization zone, (4) molding zone, (5) exit from the die.

cylinders and a screw rotated by the inner cylinder around the axis z with speed v_- and transporting the moldable mass along the specified axis with speed w_- with the stationary outer cylinder ($w_+ = v_+ = 0$).

In zone 1, the mixture is supplied to the working blades of the auger and is moved without compaction. In zone 2, the mixture is compacted. As the mixture moves toward the die, its rotation slows down, while the peripheral layers move at a faster speed. In zone 3, the density inhomogeneities arising in zone 2 are equalized due to the uneven movement of the moldable mass. Alignment is achieved by the special geometry of the blades of the point auger and their relative position. In zone 4, further alignment of the motion inhomogeneities occurs.

Mathematical modeling of the process of movement of the moldable mass in the extruder in full formulation remains a difficult task, requiring consideration of the rheological properties of the briquetted mass. Most of the known works use a simplified approach combining mathematical and physical modeling of the motion of the moldable mass. A review of modeling methods for the motion of moldable masses in an extruder is given in [10].

Consider the movement of the briquetted mass in the working chamber of the extruder from the standpoint of the basic laws of continuum mechanics and with the intention of obtaining simplified qualitative relationships that can serve as a basis for approximate calculations of the main parameters of the extrusion agglomeration process.

We assume that the mass of a wet and continuous medium rotated by the auger has an isotropic molecular pressure field p and obeys the general laws of conservation of mass, momentum and energy with proper rheology for the dynamic viscosity coefficient of the medium μ [11–16].

Particle motion is considered in cylindrical coordinates. As in [13, 14], the formalism of direct products of unit vectors and representations of matrices by double vectors (divectors) is used.

We consider the briquetted mass as a continuous medium of particles r with velocities u , with a density $\rho > 0$ described by the continuity equation

$$\rho_t = \nabla \rho u = 0$$

and pressure p (uniform Pascal stress $p\bar{e}$). Adding to the rate of change of the bulk density of the pulse ρ_u (ρu_t) its acceleration in the medium

$$\rho u \nabla u = \nabla \varepsilon u u - u \nabla \rho u = \rho_t + \nabla \rho u u,$$

and to the density ρg of a given field of a specific external force $g = g(t, r)$ —the volume density of the Archimedes force $-\nabla p = -\nabla p\bar{e}$

$$A = - \int_{\partial V} p dS = - \int_V (\nabla p) dV,$$

pushing the volume V on the area $dS = \sqrt{dS \cdot dS}$ of its boundary ∂V in the direction of the unit external normal $n = \frac{dS}{dS}$, we obtain the Euler hydrodynamic equations:

$$\rho u_t + \rho u \nabla u = \rho g - \nabla p, \text{ or } (\rho u)_t + \nabla \bar{P} = \rho g$$

$$\text{at } \bar{P} = \rho u u + p\bar{e}.$$

where arising as a force

$$\int_{\partial V} dS \bar{P} = \int_V (\nabla \bar{P}) dV,$$

tension

$$\bar{P} = \rho u u + p\bar{e} - \mu \bar{b},$$

is refined by the coefficients of dynamic and bulk viscosities $\mu = \mu(t, r) > 0$ and $\zeta\mu$, and the matrix

$$\bar{b} = \bar{\tau} - \left(\frac{2}{3} - \zeta\right)(\nabla u)\bar{e}, \quad \zeta = \text{const} \geq 0.$$

The Navier–Stokes equations in the case under consideration are as follows:

$$\begin{aligned} (\rho u)_t + \nabla \bar{P} &= \rho g \text{ for } \bar{P} = \rho uu + P\bar{e} - \mu\bar{\tau} \\ \text{and } P &= p + \mu\left(\frac{2}{3} - \zeta\right)(\nabla u) \end{aligned} \quad (1)$$

or

$$\begin{aligned} (\rho u)_t + \left\{ (\rho w u)_z + \frac{(r\rho uu)_r}{r} + \frac{\rho v^2}{r} + P_r - [\mu(u_z + w_r)]_z \right. \\ \left. - 2\frac{(\mu r u_r)_r}{r} + \frac{2\mu u}{r^2} \right\} I + \left\{ (\rho w v)_z + \frac{(r^2\rho uv)_r}{r^2} - (\mu v_z)_z \right. \\ \left. - \frac{1}{r} \left[\mu r^2 \left(\frac{v}{r}\right)_r \right] - \mu \left(\frac{v}{r}\right)_r \right\} J + \left\{ (\rho w w)_z + \frac{(r\rho uw)_r}{r} \right. \\ \left. + P_z - 2(\mu w_z)_z - \frac{[\mu r(u_z + w_r)]_r}{r} \right\} K = \rho g; \\ P = p + \mu\left(\frac{2}{3} - \zeta\right) \left[\frac{1}{r}(ru)_r + w_2 \right], \\ \rho_t + \frac{1}{r}(r\rho u)_r + (\rho w)_z = 0. \end{aligned}$$

Consider the stationary motion of a continuous medium with a constant density:

$$\rho = \text{const} > 0 \quad (\nabla u = 0) \quad \text{and} \quad \mu = \text{const} > 0,$$

in the limited section

$$0 < z < l, \quad l = \text{const} > 0,$$

of the spaces between two infinite coaxial cylinders,

$$\varepsilon a = r_{\min} < R < r_{\max} = a, \quad \varepsilon, a = \text{const} > 0 \quad (\varepsilon < 1)$$

at $-\infty < z < \infty$, internal ($r = \varepsilon a$, with the index “–”), external ($r = a$, with the index “+”), moved along the common axis $r = 0$ and rotated around it with constant speeds w_{\mp} and v_{\mp} , accordingly, under adhesion conditions

$$w|_{r=\varepsilon a, a} = w_-, w_+, \quad v|_{r=\varepsilon a, a} = v_-, v_+, \quad w_{\mp}, v_{\mp} = \text{const} \quad (2)$$

and constant pressure

$$\begin{aligned} -p_z &= \frac{p_+ - p_-}{l} = \text{const} > 0, \quad 0 < z < l, \\ p_- &= p|_{z=0}, \quad p_+ = p|_{z=l}. \end{aligned} \quad (3)$$

with specified pressure values p_{\mp} at the ends $z = 0, l$ of the section $0 < z < l$.

In the absence of mass forces and radial displacements of the rotational and longitudinally shifted medium, under the assumption that

$$\begin{aligned} u &= 0, \quad v = v(r), \quad w = w(r), \\ \text{and } g^r &= g^z = 0, \end{aligned} \quad (4)$$

its dynamic equilibrium (1)–(4) reduces to the relations:

$$\begin{aligned} -\frac{1}{r}(rw_r)_r &= \frac{-p_z}{\mu}, \quad (rv_r)_r = \frac{v}{r}, \quad p_r = \frac{\rho v^2}{r}, \\ -p_z &= \frac{p_+ - p_-}{l}, \quad \varepsilon a < r < a, \quad 0 < z < l, \\ w(\varepsilon a) &= w_-, \quad w(a) = w_+ = 0, \\ v(\varepsilon a) &= v_-, \quad v(a) = v_+ = 0. \end{aligned}$$

Dynamic equilibrium can also be reduced to a spiral flow, i.e., to the combined Hagen–Poiseuille and Couette flow [11, 12, 17],

$$\begin{aligned} w(r) &= w_+ - \frac{p_z(a^2 - r^2)}{4\mu} \\ &+ \left(w_- - w_+ + \frac{p_z a^2(1 - \varepsilon^2)}{4\mu} \right) \frac{\ln \frac{r}{a}}{\ln \varepsilon} \end{aligned} \quad (5)$$

with additional azimuthal speed

$$v(r) = \frac{\varepsilon a v_- - \varepsilon v_+}{r(1 - \varepsilon^2)} + \frac{r v_+ - \varepsilon v_-}{a(1 - \varepsilon^2)}, \quad (6)$$

and pressure

$$\begin{aligned} p(r, z) &= p_+ - \frac{z}{l}(p_+ - p_-) - \int_r^a \frac{\rho v^2(r')}{r'} dr', \\ p_r &= \frac{\rho v^2}{r} > 0, \quad \varepsilon a < r < a, \quad 0 < z < l. \end{aligned}$$

Consider the case of a fixed external cylinder:

$$w_+ = v_+ = 0, \quad w_- > 0, \quad v_- < 0. \quad (7)$$

Figure 2 shows dimensionless profiles of azimuthal and axial velocities

$$\frac{v(r)}{v_-} = Y = \frac{\varepsilon}{1 - \varepsilon^2} \left(\frac{1}{X} - X \right), \quad (8)$$

$$\varepsilon = \frac{r_{\min}}{r_{\max}} < X = \frac{r}{r_{\max}} < 1, \quad r_{\max} = a,$$

$$\frac{w}{w_p} = Z = 1 - X^2 + \frac{1 - \varepsilon^2 - \delta}{\ln \left(\frac{1}{\varepsilon} \right)} \ln X, \quad (9)$$

$$w_p = \frac{-p_z a^2}{4\mu}, \quad \delta = \frac{w_-}{w_p},$$

accordingly, with dimensionless factors ε, δ and dimensional velocity of viscous pressure w_p . The former lead to a critical parabola $\delta = 1 - \varepsilon^2$, and to the following (i) subcritical, $\delta < 1 - \varepsilon^2$, and (ii) supercritical, $\delta \geq 1 - \varepsilon^2$, spiral flow regimes:

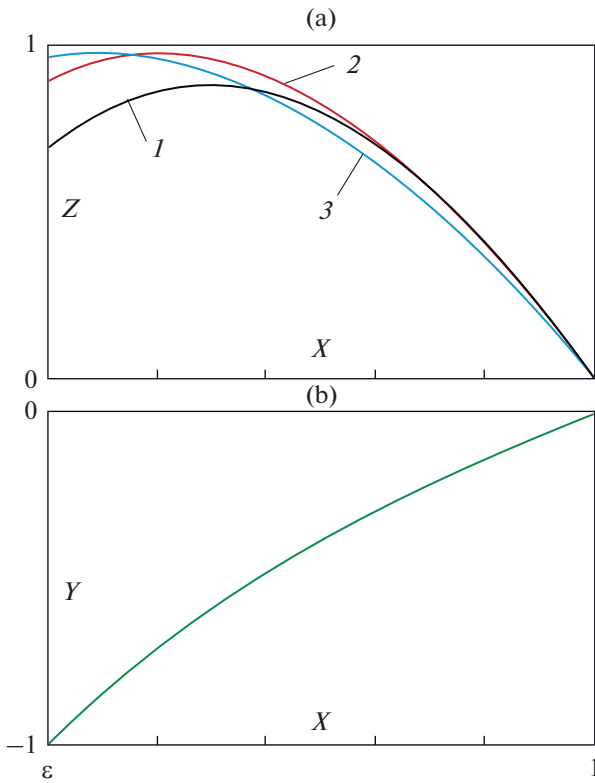


Fig. 2. Profiles of axial (left) and azimuthal (right) components of the considered spiral flow with two subcritical (1 and 2) and one supercritical (3) factor δ values.

(i) In the subcritical mode $\delta < 1 - \varepsilon^2$, the maximum possible speed of transportation, i.e. $w(r)$, longitudinal displacement, is

$$w_* = \max_{\varepsilon a \leq r \leq a} w(r) = w(r_*) = w_p Z_* < w_p,$$

$$Z_* = \max_{\varepsilon \leq X \leq 1} Z(X_*), \quad \varepsilon < \frac{r_*}{a} = X_* < 1,$$

at $\delta < 1 - \varepsilon^2$,

achieved within the flow region, $\varepsilon a < r_* < a$, and close to the velocity of viscous pressure w_p .

(ii) With a further increase in the factor δ , the value w_* reaches its maximum possible value w_p at the inner boundary

$$w_* = w(r_*) = w_p Z_* = w_p, \quad Z_* = 1,$$

$$\frac{r_*}{a} = X_* = \varepsilon, \quad \text{at } \delta \geq 1 - \varepsilon^2,$$

and remains the maximum possible speed of the transported mass $w_* = w_p$ at any supercritical factor δ (Fig. 2).

It can be shown that for the case of a stationary outer cylinder, there is a critical rotation speed of the inner cylinder,

$$w_c = (1 - \varepsilon^2) w_p \quad (\text{or } \delta = 1 - \varepsilon^2),$$

such that, with the appropriate subcritical or supercritical mode, the maximum transport speed is reached between the cylinders or on the inner cylinder and is less than or equal to the speed of the viscous pressure

$$w_p = \frac{-p_z a^2}{4\mu},$$

respectively:

$$w(r_*) < w_p \quad \text{and } \varepsilon a < r_* < a$$

$$\text{at } w_- < w_c \quad (\text{or } \delta < 1 - \varepsilon^2),$$

$$w(r_*) = w_p \quad \text{and } r_* = \varepsilon a$$

$$\text{at } w_- \geq w_c \quad (\text{or } \delta \geq 1 - \varepsilon^2).$$

Consider the case of compressibility of the moldable mass:

$$\rho = \rho(r) \neq \text{const} \quad (\nabla u \neq 0) \quad \text{and } \mu = \text{const}.$$

Under the assumption that

$$u = u(r), \quad v = v(r), \quad w = w(r) \quad \text{and } g^r = g^z = 0,$$

the continuity equation in (1) leads to a constant

$$r\rho u = m = \text{const}, \quad \text{or } ru = m\varphi,$$

where $\varphi = \frac{1}{\rho}$ – specific volume. Its relation to dynamic

viscosity $\left(\alpha = \frac{m}{\mu}\right)$ is a dimensionless quantity. Putting in (1)

$$p_r = \frac{\rho v^2}{r} \quad \text{and} \quad \frac{\alpha + 1}{r^2} (ru)_r$$

$$- \left(\frac{1}{3} + \zeta\right) \left(\frac{1}{r} (ru)_r\right)_r - \frac{1}{r} (ru)_{rr} - \frac{\alpha ru}{r^3} = 0,$$

we find for given values φ_{\mp} :

$$\varphi = \frac{\varepsilon^{\lambda_+} \varphi_+ - \varphi_-}{\varepsilon^{\lambda_+} - \varepsilon^{\lambda_-}} \left(\frac{r}{a}\right)^{\lambda_-} + \frac{\varepsilon^{\lambda_-} \varphi_+ - \varphi_-}{\varepsilon^{\lambda_-} - \varepsilon^{\lambda_+}} \left(\frac{r}{a}\right)^{\lambda_+},$$

$$\lambda_{\mp} = \frac{8 + 6\zeta + 3\alpha}{2(4 + 3\zeta)} \mp \sqrt{\left(\frac{8 + 6\zeta + 3\alpha}{2(4 + 3\zeta)}\right)^2 - \frac{3\alpha}{4 + 3\zeta}}$$

$$\text{at } \left(\frac{8 + 6\zeta + 3\alpha}{2(4 + 3\zeta)}\right)^2 > \frac{3\alpha}{4 + 3\zeta},$$

Satisfying conditions (2)–(3) and equations (1),

$$\frac{m}{r} v_r + \frac{mv}{r^2} - \frac{\mu}{r} (rv_r)_r + \frac{\mu v}{r^2} = 0,$$

$$\frac{m}{r} w_r - \frac{mw}{r^2} - \frac{\mu}{r} (rw_r)_r = -P_z = -p_z,$$

$$P = p + \mu \left(\frac{2}{3} - \zeta\right) \frac{1}{r} (ru)_r, \quad \varepsilon a < r < a,$$

the axial and azimuthal velocity components in this case have profiles:

$$w = \frac{\varepsilon^{\gamma_+} w_+ - w_-}{\varepsilon^{\gamma_+} - \varepsilon^{\gamma_-}} \left(\frac{r}{a}\right)^{\gamma_-} + \frac{\varepsilon^{\gamma_-} w_+ - w_-}{\varepsilon^{\gamma_-} - \varepsilon^{\gamma_+}} \left(\frac{r}{a}\right)^{\gamma_+} + \frac{-p_z a^2}{\mu(4-\alpha)} \left(\frac{\varepsilon^{\gamma_+} - \varepsilon^2}{\varepsilon^{\gamma_+} - \varepsilon^{\gamma_-}} \left(\frac{r}{a}\right)^{\gamma_-} + \frac{\varepsilon^{\gamma_-} - \varepsilon^2}{\varepsilon^{\gamma_-} - \varepsilon^{\gamma_+}} \left(\frac{r}{a}\right)^{\gamma_+} - \left(\frac{r}{a}\right)^2\right),$$

$$\gamma_{\mp} = \frac{\alpha}{2} \mp \sqrt{\left(\frac{\alpha}{4} - 1\right)\alpha}$$

at $\left(\frac{\alpha}{4} - 1\right)\alpha > 0$,
and

$$v = \frac{\varepsilon^{\beta_+} v_+ - v_-}{\varepsilon^{\beta_+} - \varepsilon^{\beta_-}} \left(\frac{r}{a}\right)^{\beta_-} + \frac{\varepsilon^{\beta_-} v_+ - v_-}{\varepsilon^{\beta_-} - \varepsilon^{\beta_+}} \left(\frac{r}{a}\right)^{\beta_+},$$

$$\beta_{\mp} = \frac{\alpha}{4} \mp \left(\frac{\alpha}{2} + 1\right) = -\frac{\alpha}{4} - 1, \frac{3\alpha}{4} + 1.$$

Thus, the axial and angular (azimuthal) speeds of the spiral flow are determined respectively by the speeds of transportation and rotation by the auger of the mixed mass, w and v , respectively. In contrast to dry bulk material (from small and solid particles) [18], this mass is a pasty mass of a viscous-plastic continuous medium, in which the force field of contact stresses $\bar{\tau}$ is beyond the plasticity limit, where the Bingham rheology [14, 15] reduces to constant viscosity, i.e. approaching Newtonian rheology, as in [19–21].

CONCLUSIONS

A stationary spiral flow with constant or variable density satisfactorily describes the movement of the briquetted mass in the extruder.

There exists the highest possible speed of transportation (speed of viscous pressure w_p).

The obtained dependences are also applicable to the calculation of extruders “semi-stiff” and “soft extrusion”.

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